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AUTHOR Hillegeist, Eleanor; Epstein, Kenneth
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ABSTRACT

The study examined the relationship between language and mathematics with 11 classes of deaf students taking Algebra 1 or Algebra 2 at the Gallaudet University School of Preparatory Studies. Specifically, the study attempted to predict the difficulty of a variety of relatively simple algebra problems based on the abstractness of the math and the amount of English required. Types of equations and problems were categorized into five categories based on mathematical abstractness and English requirements. The study found that difficulty levels across groups consistently showed the following sequence: concrete, If/Then--concrete, abstract, If/Then--abstract, and If/Then--abstract plus extraneous English information. Somewhat surprisingly, the If/Then English construction was less difficult than substituting an alphabetic variable for a number. Problems with defining signs and mathematical terms are noted as are approaches to the use of space to explain concepts. The importance of developing an adequate language of instruction to help students integrate the semantic and syntactical aspects of mathematics and language is stressed. (DB)

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INTERACTIONS BETWEEN LANGUAGE AND MATHEMATICS WITH DEAF STUDENTS: DEFINING THE "LANGUAGE-MATHEMATICS" EQUATION

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Eleanor Hillegeist
Kenneth Epstein

Gallaudet University
School of Preparatory Studies

Abstract

Experience at the School of Preparatory Studies at Gallaudet University clearly shows that as mathematical concepts become increasingly complex and the language of mathematics becomes increasingly precise, the success of the teaching-learning process declines. Thus, students show marked improvement in Algebra 1 over the course of a semester or a year, but much smaller improvement in Algebra 2 or Geometry. While there is no question that the increase in difficulty of the mathematical concepts is an important factor in these observations, there also seems to be an effect specifically related to language.

At this point, the nature of the language effect is not completely clear. The property of sign language which tends to associate specific signs with specific concepts is important. The special role of language in mathematics which assigns specific meanings to everyday words is also a factor. The lack of a standard mathematical sign vocabulary among deaf and hearing faculty and students is a third area of concern. Often, all three factors combine to make understanding very difficulty. For example, words such as "consistent," "function," "direct," or "indirect," which have common everyday meanings and concept signs associated with those meanings have different specific mathematical meanings. A mathematics student need only know the specific mathematical meanings and definitions to apply them to solving mathematical problems. However, without knowledge of the mathematical meanings or if the mathematical meanings become confused with the common meanings, mathematics may be extremely difficult if not impossible to learn.

None of the research on the relationship between language and mathematical proficiency includes sign language and deaf students. Moreover, the research indicates that problems are not consistent across all language minority students and that the same instructional interventions may not be appropriate for all groups.

The intent of the first of our studies was to predict the difficulty of a variety of relatively simple algebra problems based on the "abstractness" of the math and the amount of English required. We operationally defined "abstractness" in terms of the right side of an equation. Equations which had numbers on the right of the equal sign were defined as less abstract than those that had alphabetic character variables on the right of the equal sign. For example, "x^2 = a" is more abstract than the otherwise equivalent "$x^2 = 9$." We defined "amount of English" in terms of the language surrounding the equation. For example, "If $x^2 = 25$, then x = ____" requires more English than the otherwise equivalent "$x^2 = 9$." Finally, problems varied in mathematical difficulty. For example, "$(x+y)^2 = 36$" was considered more difficult than "$x^2 = 9$."

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The test included 25 problems arranged in five groups of five problems each. Problems within each group become systematically more difficult, and were kept parallel across groups. Each of the groups varied in abstraction and amount of English necessary. Thus, the first group was "Concrete (C)," the second "Abstract (A)," the third "If/then - concrete (ITC)," the fourth "If/then - abstract (ITA)," and the last group was "If/then - abstract with additional extraneous English information (ITA+)."

The first analyses focused on patterns of item difficulties. difficulty levels across groups consistently show the pattern "C - ITC - A - ITA - ITA+." Somewhat surprisingly, the "If/then" English construction was less difficulty than substituting an alphabetic variable for a number. Difficulties within groups are not as consistent, but one item tended to be most difficult. That item was of the form "$nx^2 = 36$." Again, the alphabetic variable caused unexpected difficulty for the students.

Introduction

The Gallaudet University School of Preparatory Studies serves students who do not have the mathematical and English skills necessary to begin the University's undergraduate liberal arts curriculum. In each year's class more than half are not proficient in elementary high school algebra. Less than half have a good understanding of basic algebra, yet do not understand second year algebra. The students not prepared for fundamental algebra are deficient in basic computational skills and have a very weak understanding of fundamental mathematical concepts. Few of these students have mastered geometry.

The poor level of mathematical skills of most new preparatory students is not surprising. However, it is disturbing to note the pattern of success for students attending classes at the School. Most beginning Algebra 1 students do quite well. Algebra 2 students do not do as well. Geometry students perform least well of the three groups. What is the explanation for the relative lack of success of the more advanced students? We believe that the explanation involves a combination of the increasing abstractness and complexity of the mathematical concepts and the difficulty of finding an effective language in which to teach and learn those concepts.

In fact, there are three "languages" that must be considered. These are the *language of mathematics*, the *language of instruction*, and the *language of learning*. The language of mathematics is a highly precise, specialized language. It includes technical terms, symbols, syntax, figures, graphs, relational statements, and usage of numbers and words in a manner unique to mathematics. Mathematical language lacks the redundancy of natural spoken, written or signed languages. Algebra is an example of a nonredundant language rich communication system. For example, in $2(x + y) -3x + 5y$ the basic elements occur almost equally often and are subject to minimal sequential restraints. Lost or distorted symbols cannot easily be reconstructed. Students accustomed to communicating in a highly redundant language find it difficult to adapt to the language systems of advanced mathematics. Thus the language of mathematics must be explicitly taught and learned.

The language of instruction is the language used by the instructor and the language used in textbooks. Textbook instruction uses English and the language of mathematics. Actual classroom instruction is probably delivered in some combination of English and American Sign Language (ASL).

The language of learning is the language used by each student. The most outstanding characteristic of the student population is the enormous variation in the language skills represented. The language of learning may be ASL, English, a foreign language, a combination of ASL and English, or some non-systematic variety of English. Success in learning mathematics depends upon how successful each student is in integrating the language of mathematics, the language of instruction, and his or her own language of learning.

We believe that as mathematical concepts and procedures become increasingly abstract and complex, the importance of language and communication increases. As the mathematics becomes more difficult, differences in the language of instruction and the language of learning

will tend to interfere with learning. Mismatches in the language of instruction and the language of learning may be an important part of the reason that Algebra 2 instruction tends to be less successful that Algebra 1 instruction at the School of Preparatory Studies.

In this paper we consider the literature on learning mathematics and the interactions between language and mathematical skills. We follow the literature review with the results of a pilot study to investigate the effect of the levels of complexity and abstraction in English and the language of mathematics on student performance. The final section of the paper discusses what the literature and observations suggest about why it is so difficult to effectively teach mathematics to college-age deaf students and what must be changed to increase the effectiveness of instruction.

Review of Literature

Learning Mathematics

Resnick etal. (1987) provide a cognitive approach to understanding how students learn algebra. Their article begins with the important reminder that "Number is a strictly cognitive entity. People construct this cognitive entity, the concept of number, without benefit of any physical numbers to inspect or analyze...people must reason about objects that exist only as mental abstractions (p. 170)." Algebraic symbols and rules comprise a formal system for communicating mathematical concepts and describing actual events. The generalizability of mathematical rules is what makes mathematics a powerful tool.

The literature on learning mathematics and the Goldberg and Costa (1981) explanation of basic processes involved in acquiring, organizing and accessing knowledge provide support for the concern with language and communications problems (1981). Goldberg and Costa suggest that we use descriptive systems, composed of units of encoding or rules of transformation to process information. One such descriptive system, used by only a few members of all language

users is mathematics. Mathematics is difficult because it deals with phenomena that exist only as mental abstractions and because its rules require acquisition of a descriptive system that may be difficult to explain using natural language. Idiosyncratic descriptive systems used by individuals to acquire new knowledge, are unique to each individual learner. Idiosyncratic descriptive systems that individuals develop to cope with the difficulties of mathematics may incorporate distortions of the formal system of mathematics. The possibility of such distorted descriptive systems suggests the need for communication that is clear and unambiguous.

Language, Mathematics and Deafness

The body of literature focusing specifically on the relationship between language skills and mathematics learning is relatively small. However, the conclusions consistently support the importance of studying this relationship, especially for cases where there may be questions about the match between the language of learning and the language of instruction. Spanos etal. (1988) reviewed studies involving both monolingual and bilingual students and mathematics instruction. The studies suggest a close relationship between language proficiency and mathematics achievement. Spanos etal. conclude that students must achieve a minimum level of cognitive knowledge of mathematical concepts and how the concepts are applied and embedded in a language structured to express that knowledge (pp. 221-222).

Spanos etal. have classified the syntactic, semantic, and pragmatic features of language that seem to be critical to successful mathematics achievement (pp. 224-232). Syntactic features can be confusing or misleading if they are not properly understood. They are often identified with errors that appear to indicate that the student focused on the surface structure of a mathematical statement rather than on its meaning. For example, equations may be written according to the order in which quantities are mentioned in a word problem, regardless of the quantitative relationships expressed in the problem statement. A second serious syntactic

problem involves comparatives and prepositions, as in "There are five times as many men attending the university as women".

Difficult semantic features include vocabulary, meaning of words, phrases and notation, and ambiguities in directions or problem statements. Technical meanings for common words can be particularly troublesome if students have difficulty separating the common meaning from the specific mathematical meaning. Words like *function, irrational, column,* and *table* have technical definitions in mathematics which must not be confused with their common meanings. The problem is compounded with phrases comprised of several familiar words, as in "Graph the rational function".

Pragmatic features are epistemological and textual. Epistemological features relate to the knowledge and experience of the students relative to the mathematics problems they are asked to solve. Students unfamiliar with the American economic system, marketing practices or scientific laws and relationships cannot understand problems posed in these contexts. Textual features relate to the connection between mathematics instruction and students' interests and goals.

The Spanos etal. conclusions are based on studies with native English speakers as well as English-as-a-Second Language (ESL) students. Problems encountered by native speakers are compounded for ESL students because the language related problems in learning mathematics tend to be the same kinds of problems which non-native speakers face in trying to learn English. Our experience is that because many deaf students are NOT native English users, it is often appropriate and worthwhile to consider the ESL literature.

Studies of language minority students have shown that the gap in quantitative and analytical reasoning between those students and language majority students accumulates throughout the school years. While there is no evidence to suggest that the basic abilities of

language minority or bilingual students differ from English speaking students in the United States, the mathematical achievement differences between the language minority and language majority students are pronounced (Cocking and Chipman, 1987). DeAvila (1988) has proposed, in relation to Hispanic students' performance, that the heavy emphasis placed on oral language mastery may interfere with mathematical learning. Given his proposition, one must consider the possible adverse effects on learning mathematics that accompanies the demands on deaf students to learn English. Stone (1988) reports that experience at the National Technical Institute for the Deaf (NTID) clearly demonstrates the problems that deaf students have understanding the language of mathematics. The problem is accentuated when students are required to apply their mathematical skills to another content area.

The literature related to spoken languages clearly shows that conflicts between the dominant language of the student, the language of instruction, and the language in which performance is tested have an important effect on measured performance and student abilities (Cossio, 1977; Macnamara, 1967). Research by Bloomquist and Allen (1988) compared Stanford Achievement Test items for hearing and deaf students. Patterns of item difficulties differ for the two groups for math applications items. Applications problems are language dependent and tend to be disproportionately difficult for deaf students.

Research with hearing-impaired individuals suggests that observed deficits in cognitive performance may be due to experiential and linguistic deficits, communication handicaps, and secondary handicaps rather than basic cognitive abilities (Levine, 1976; Martin, 1985). Zweibel and Mertens (1984) found no differences in patterns of performance on certain cognitive tasks among deaf and hearing individuals if one allows for delayed development in the hearing-impaired population. In summarizing presentations on cognitive processes and strategies from the first International Symposium on Cognition, Education and Deafness, Wolff (1985) suggests

that researchers remain sensitive to linguistic and modality specific considerations when studying cognitive processing of the hearing impaired. Apparent cognitive-processing problems may be linguistic-conceptual differences rather than an inherent inability to think abstractly.

What, then, is the relationship between language and abstract and complex mathematical thinking? The literature clearly shows mathematics to be a cognitively-demanding discipline. Relatively good language skills appear to be necessary to successfully learn mathematics, and differences in the language of instruction and the language of learning can interfere with the teaching and learning process. However, the evidence clearly shows that deaf individuals do have the ability to reason abstractly. Therefore, deaf college students should be able to learn algebra and more advanced mathematics if the basic skills are in place and if the more advanced topics are presented properly. In order to identify effective instructional strategies, we believed that we needed a better understanding of the role of complexity and abstraction in language and mathematics in mathematics learning. Thus, we turned to classroom experiences to begin our investigation.

Classroom Observations

Rationale

Students often do not understand what is expected of them, they have difficulty expressing mathematical concepts in English or in sign, and they find mathematics difficult. For example, many students cannot explain the meaning of directions such as "Evaluate," "Solve the system," or "Specify the domain and range of the function." Still fewer understand that the process of "solving" a quadratic equation is the same as "finding its roots." Other classroom observations led to questions about the relationship between language and understanding. For example, students find the following mathematical statement difficult to understand.

1) If x is a real number, $|x|$, the absolute value of x, is defined by

$$|x| = \begin{cases} x, \text{ if } x >= 0. \\ -x, \text{ if } x < 0. \end{cases}$$

Students also demonstrate difficulty with manipulating more than one variable. For example, while students can solve a linear equation in one variable (e.g. $x + 4 = 11$), they have difficulty solving more complex literal equations such as $P = 2L + 2x$ for x. Similar difficulties occur with quadratic equations and when variable coefficients and terms are substituted for concrete numeric values. Observations of students struggling with these kinds of problems, together with the knowledge that many of our students are not successful in Algebra 2, led us to wonder about the impact of abstraction and complexity in language and mathematics on learning algebra.

Data Collection

Data were collected during the spring and summer of 1988. Three classes of Gallaudet University School of Preparatory Studies summer- school algebra students responded to a test requiring them to solve simple linear equations (Figure 1). These students were taking Algebra 1 or Algebra 2 and had completed instruction on the concepts and skills being tested. Eight classes of Spring 1988 algebra students responded to a test requiring them to solve quadratic equations (Figure 2). Five of these classes were Algebra 2 classes and had completed instruction on the concepts and skills being tested. The other three classes were Algebra 1 classes. These students, while having received instruction in solving quadratic equations by taking the square root, may not have solved problems of the same complexity and abstractness as those included on the test.

All testing was conducted during regular class periods. The time limit for the test was 45 minutes, and all students comfortably completed the test within the time limit. Of the instructors participating in the study, five were deaf and three were hearing. They explained the test to the students in sign language. They were asked not to answer any student questions during the test. In particular, they were to answer no questions related to language. If a student did not know what to do with those parts of the test in which the basic mathematical question was embedded in a statement, the instructor did not explain what was expected. Each student had to interpret the question for himself or herself.

Analysis of the Data

For the purpose of this investigation, mathematical expressions containing variable terms and coefficients were defined to be more abstract than those composed of numeric terms or coefficients. Language abstraction and complexity increased as the directions increased from simple statements to three word conditional sentences to conditional sentences containing a clause.

The linear equations test consists of four groups of four items each (Figure 1). The language used for Groups I and II is less abstract than that used for Groups III and IV. The mathematics in Groups I and III is less abstract than that represented in Groups II and IV. Mathematical complexity is not systematically varied in the linear equations test since each problem requires only one operation for its solution. By combining the variation in language and mathematical abstraction, it is possible to characterize and name each of the four groups of problems as follows. Group I, representing concrete language and mathematics, is designated C/C. Group II, representing concrete language and abstract mathematics, is designated C/A. Group III, designated A/C, represents abstract language and concrete mathematics. Finally,

Group IV represents abstract language and mathematics and is designated A/A. Similar addition, subtraction, multiplication, and division problems are included in each group.

A similar categorization scheme is used with the quadratic equations test (Figure 2). Groups I through IV on the quadratic test represent the same combinations of language and mathematics abstraction as the linear equations test. Group V adds an additional phrase to the problem statement, making the language more complex. Hence, Group V will be designated A+/A. In addition, there are differences in the mathematical complexity and abstraction among the items within each group. The third item in each group (e.g. $nx^2 = 81$ in group I) substitutes a variable coefficient for the numeric coefficient in the otherwise identical second problem in each group (e.g. $4x^2 = 25$ in group I). The fourth and fifth items in each group require two operations in their solutions. Therefore, they add a level of mathematical complexity. Finally, the fifth problem in each group substitutes a variable term for a comparable numeric term in the fourth problem of each group, adding to the mathematical abstractness of the problem.

Test results are reported in Tables 1 and 2. The Blank, Wrong, Sign, and Correct columns each show the number of students responding in each category. A blank, or no response, was scored 0. A computationally incorrect response received a score of 1. A computationally correct response with a sign error, on the quadratic test, was scored 2. A completely correct response received a score of 2 on the linear equations test and 3 on the quadratic equations test. The *Score* column is a computed item easiness score which equals the sum of the item response scores across examinees. It provides a finer level of discrimination than a simple number correct score. Comparing the *Correct* column with the *Score* column shows that items that would be considered equivalently easy, based on the number of students responding correctly, are different when the comparison is based on the computed score. For

example, items 1 and 2 on the quadratic equations test were answered correctly by 22 students each. However, many more students left item 2 blank (22 versus 6), more gave a totally incorrect answer to item 2 (31 versus 25), and more answered item 1 correctly except for a mathematical sign error (34 versus 12). Hence, the computed score indicates that item 1 is easier than item 2. In most cases, the overall interpretation of trends in the data based on the computed score is no different than that based on number correct. However, we believe that the added sensitivity made possible with the computed score is valuable in the kind of micro-analysis required by this study.

We considered the results in three ways. First, we sought trends based on the difficulty of the items considered as a single group. Second, we compared the difficulties of different item types within each group defined by the level of language and mathematical abstraction. Third, we considered item difficulties of similar item types across abstraction groups.

The most difficult items on the linear equations test were numbers 16 and 8. These were division problems representing a higher level of mathematical abstraction. The next most difficult were numbers 15 and 7. These were multiplication problems, also representing the higher level of mathematical abstraction. Five problems were the next most difficult (Items 14, 6, 13, 5, and 12). They represented the other items in the mathematically abstract group and the division problem in the mathematically more concrete, but relatively abstract language group. The remaining division problem (Item 4) was the next most difficult problem. All of the other problems were relatively easy.

Table 3A compares the item difficulties on the linear equation test within abstraction groups. For example, the first line of the table shows that, within the relatively concrete language and mathematics group (Group I, C/C), item types A and B were equally easy, item type C was next hardest, and item type D was the hardest. One is struck by the consistency of

the results. Item type D was universally the hardest, followed by item type C. Item types A and B were uniformly easy.

Table 3B compares the item difficulties for each item type across language/mathematical abstraction groups. For example, column 1 indicates that item type A was equally easy in the concrete language/ concrete mathematics group (Group I, C/C) and the abstract language/ concrete mathematics group (Group III, A/C). Item type A was more difficult when it appeared in the more mathematically abstract groups (Group II, C/A and Group IV, A/A). This table is also remarkably consistent. Regardless of the item type, the relative language abstraction is less important than the relative mathematical abstraction. The more mathematically abstract groups (Groups II and IV) were more difficult than the more mathematically concrete groups (Groups I and III). If there were a systematic language effect, we would have expected Groups I and II to be easier than the mathematically equivalent Groups III and IV. The data do not show those differences.

Considering the three views of the data as a group, the trends are clear. Division and multiplication problems are more difficult than addition and subtraction problems. Mathematically more abstract items, which substitute variable coefficients and terms for numeric coefficients and terms, are more difficult than more mathematically concrete items. Mathematical abstraction is a more potent source of difficulty than language abstraction. In fact, for this test, the presence of a conditional if/then statement had little effect on the difficulty of the items.

The most difficult items on the quadratic equations test were the last five. These were the items representing the most complex and abstract language on the test. These results are different than the linear equations test results in which the level of language abstraction was

relatively unimportant. However, the language used for these items was considerably more difficult than any of the language used on the linear equations test.

The next two most difficult items were "If $nx^2 = J$, then $x =$ ____." (item 18) and "$nx^2 = c$" (item 8). These items were both mathematically abstract and complex. Item 18, in the more complex language group of problems, was slightly more difficult than item 8. The order of the remaining items was not perfectly unambiguous, but the tendency was that the more mathematically abstract and complex items were more difficult than the less mathematically abstract and complex items expressed by more abstract language. The easiest items appeared among the less mathematically complex and abstract items. In other words, the relatively greater importance of mathematical abstraction and complexity observed in the linear equations test results reappeared.

Table 4A compares the item difficulties on the quadratic equations tests within abstraction groups. For example, the first line of the table shows that, within the relatively concrete language and mathematics group (Group I, C/C), item type A was the easiest, followed by item types B, D, E, and C which was the hardest. Looking down the table, one can see that item type A is uniformly the easiest and item type C was most often the hardest. While there are some reversals in the table, item type E tended to be comparatively difficult and item types B and D were relatively easy. Thus, the order of mathematical difficulty was relatively independent of the complexity and abstractness of the language used to present the problem.

Table 4B compares the items difficulties for items of the same mathematical type across abstraction groups. For example, the first column shows that item type A was easiest in the concrete language/concrete mathematics group (Group I, C/C) and hardest in the abstract and complex language/abstract mathematics group (Group V, A+/A). The abstract language/concrete mathematics group (Group III, C/A) was the second easiest, followed by the

concrete language/abstract mathematics group (Group II, C/A) and the abstract language/abstract mathematics group (Group IV, A/A). In other words, with the exception of the most complex and abstract language group (Group V, A+/A), mathematical abstractness was more likely to make an item difficult than abstract language. These results hold for all item types and are consistent with the results found on the linear equations test.

These data clearly show systematic patterns of item difficulties. They suggest that the "If/then" language construction can either be ignored or that it is relatively easy compared with the effect of replacing numbers with variables in these types of problems. However, the results of the quadratic equations test do begin to hint at a language effect, especially when the language becomes complex enough to include clauses as well as conditional statements. The results suggest that there may be a threshold level of language complexity and abstractness that seriously interferes with the students' abilities to solve mathematics problems. Further studies should attempt to define such thresholds for both language and mathematics.

The test data demonstrate in a structured and controlled manner a small part of an overall pattern observed in mathematics classrooms. Students have problems dealing with the rule-governed, symbolic language of mathematics. The most difficult test items were those that were more representative of formal mathematics language. The problems of understanding become more acute as the complexity and abstractness of the mathematics increases. A number of factors may be contributing to this result at the School of Preparatory Studies.

Students vary a great deal in their "native" languages. For those students who have grown up as members of the deaf community, their language is likely to be American Sign Language (ASL). Other students seem to behave as if their native language is English. These are students who lost their hearing after learning English, or who grew up in oral environments, or who somehow learned English as a native language. A third group of students who display

language behaviors that are very difficult to classify. These students tend to be the only deaf member of their families, their hearing impairment may or may not have been diagnosed early, they tend to have "mainstreamed" educational backgrounds, and they display highly individualistic and idiosyncratic language. All of these language types are likely to be present in any given classroom.

Instructors do not share a common sign language for mathematical terms. Hence there is a continuous need to define signs and terms. The definition of terms is further complicated by the need to choose among alternate possible signs and by the differences between everyday meanings of signs and technical mathematical meanings that must become associated with those signs. It is particularly difficult because the teacher often does not know all of the common meanings and is almost certain not to know particular regional meanings attached to signs. Many of the students do not appear to have either the sign language or a written/spoken language to effectively learn mathematics and solve mathematics problems. They seem to have limited life experiences upon which to draw to derive meaning from new concepts or to fully understand example problems.

Use of Space

The use of space, spatial relationships, and signed explanations seems to have great value in teaching mathematical ideas. Students may be able to describe and solve an algebra word problem in sign, yet be unable to write it or to solve similar written problems. Geometry concepts also seem to require spatial representations. It is important to note that the spatial representations are not necessary formal sign language explanations. We believe that one of the critical features of sign language, whatever combination of ASL, English, gesture, or fingerspelling one chooses, is that it permits one to use space to explain concepts. Students

seem to be able to learn and explain mathematical ideas much more effectively when they can use signs and space than when they are forced to read or write about them.

Speculations and Implications

The literature and our experience tell us that mathematics is a difficult topic for many students. Mathematics is concerned with abstractions and not with everyday objects; it requires reasoning and analysis skills. The mathematical teaching and learning process is heavily dependent on effective communication, requiring precise language to convey mathematical ideas. Excellent language understanding is required to interpret problems properly and proper use of technical vocabulary is a necessary part of mathematical problem solving.

Deafness adds special considerations to teaching and learning mathematics. Differences between signed and spoken languages complicate communication. The extremely varied language backgrounds of both students and mathematics instructors create impediments to effective communication. We believe that deaf students take advantage of spatial information to improve their understanding of complex ideas and that effective use of space is critical to effective communication.

Goldberg and Costa's explanation of descriptive systems leads to the speculation that students may have very non-traditional and often incorrect understandings of mathematics which are internally consistent from the point of view of the student. This possibility seems more tenable when one considers that the pre-college experiences of many of our students may not have included clear and effective communication.

What are some implications for research and instruction? Clearly, we must be more precise in describing students' mathematical behavior. We need more structured research to identify particular conceptual problems common to many students. We must identify the interaction of spoken, written, and sign language in communicating mathematics to deaf stu-

dents. We need to know more about cognitive structures, whether they are descriptive systems, schemata, semantic networks, or one of the other currently debated explanations of the cognitive processing which underlies our students' mathematical behavior. How do we apply concepts of general cognitive psychology to understand how deaf students process sequential and spatial materials to access the concepts of higher mathematics? We need a cooperative relationship between mathematics educators and researchers in communication, cognitive science, linguistics and mathematics to understand the cognitive components of the interaction of the several language systems involved in the teaching and learning process.

With respect to instruction, it is important to make the results of research accessible to teachers. The gap between classroom applications and cognitive, communication, mathematics, and educational research is too large. Researchers must consider and explore the potential instructional implications of their work. Instructors, open to the findings of researchers, must begin to describe and analyze the features of appropriate mathematical behavior. They must develop teaching strategies that reflect understanding of the integral relationships between the context, the language of mathematics, and the cognitive factors in communication that are required to meet the communication needs of hearing-impaired students.

Evidence from the literature comparing the four common receptive communication modalities suggests that deaf students perform most effectively when provided with written information, despite their often poorly developed grammatical skills (Grove and Rodda, 1984). Lipreading appears to be the least effective of all the modalities; manual and total communication modalities come between lipreading and reading. Mathematics instructors must find ways to exploit these findings. They need to use and develop visually and spatially rich instructional environments that facilitate access to mathematics. An adequate language of instruction must be developed to help students integrate the semantic and syntactical aspects of mathematics and

language to access the deeper structural aspects of higher mathematics. Finally, we need increased emphasis on the design of testing and assessment activities that do not hinder student performance because of dependence on the syntactic aspects of spoken written language.

We believe, that there must be a much greater emphasis on language and communication in mathematics classes. The traditional assumption that students will learn their mathematics if it is only repeated and practiced enough, must be questioned. Students may not be able to understand everything we sign or say, and it will require cooperation on the part of instructors, cognitive scientists, mathematicians, linguists, and experts in sign language to significantly improve communication, instruction, and learning in mathematics classrooms.

Table 1: Response patterns on linear equations test
(Scoring system: blank = 0, wrong = 1, correct = 2)

Item	Blank	Wrong	Correct	Score
1	0	0	16	32
2	0	0	16	32
3	0	1	15	31
4	0	3	13	29
5	1	4	11	26
6	1	4	11	26
7	2	4	10	24
8	3	4	9	22
9	0	0	16	32
10	0	0	16	32
11	0	0	16	32
12	2	2	12	26
13	2	2	12	26
14	2	2	12	26
15	3	2	11	24
16	3	4	9	22

Table 2: Response patterns on quadratic equations test (Scoring system: blank = 0, wrong = 1, sign error = 2, correct = 3)

Item	Blank	Wrong	Sign	Correct	Score
1	6	25	34	22	159
2	22	31	12	22	121
3	16	61	1	9	90
4	14	63	0	10	93
5	19	52	8	8	92
6	28	39	7	13	92
7	33	40	4	10	78
8	35	39	5	8	73
9	34	34	7	12	84
10	36	34	7	10	78
11	15	17	36	19	146
12	29	32	11	15	99
13	23	53	3	8	83
14	26	49	0	12	85
15	30	36	11	10	88
16	36	28	9	14	88
17	39	31	5	12	77
18	39	34	4	10	72
19	37	32	6	12	80
20	40	30	6	11	75
21	57	17	3	10	53
22	58	18	3	8	48
23	59	17	2	9	48
24	58	17	4	8	49
25	60	15	4	8	47

Table 3A: Rank order of item difficulties (1 = easiest item) within abstraction groups for the linear equations test. (Read across)

Item type:	A	B	C	D
Group				
Group I (Concrete/Concrete)	1.5	1.5	3	4
Group II (Concrete/Abstract)	1.5	1.5	3	4
Group III (Abstract/Concrete)	1.33	1.33	4	1.33
Group IV (Abstract/Abstract)	1.5	1.5	3	4

Table 3B: Rank order of item difficulties (1 = easiest item) for given item types across abstraction groups.

(Read down)

Item type:	A	B	C	D
Group				
Group I (Concrete/Concrete)	1.5	1.5	2	1
Group II (Concrete/Abstract)	3.5	3.5	3.5	3.5
Group III (Abstract/Concrete)	1.5	1.5	1	2
Group IV (Abstract/Abstract)	3.5	3.5	3.5	3.5

Notes

1. Examples of the item types in their most concrete mathematical form follow.

Type A: $x + 7 = 5$ Type B: $x - 9 = 10$

Type C: $4x = 12$ Type D: $\frac{x}{2} = 8$

2. Groups are defined on the basis of the level of language and mathematical abstraction represented by the directions and the use of numeric and variable terms and coefficients in the items. For example, Group III represents abstract language and concrete mathematics.

Table 4A: Rank order of item difficulties (1 = easiest item) within abstraction groups for the quadratic equations test.
(Read across)

Item type:	A	B	C	D	E
Group					
Group I (Concrete/Concrete)	1	2	5	3	4
Group II (Concrete/Abstract)	1	3.5	5	2	3.5
Group III (Abstract/Concrete)	1	2	5	4	3
Group IV (Abstract/Abstract)	1	3	5	2	4
Group V (Abstract +/Abstract)	1	3.5	3.5	2	5

Table 4B: Rank order of item difficulties (1 = easiest item) for given item types across abstraction groups.

(Read down)

Item type:	A	B	C	D	E
Group					
Group I (Concrete/Concrete)	1	1	1	1	1
Group II (Concrete/Abstract)	3	3	3	3	3
Group III (Abstract/Concrete)	2	2	2	2	2
Group IV (Abstract/Abstract)	4	4	4	4	4
Group V (Abstract +/Abstract)	5	5	5	5	5

Notes

Examples of the item types in their most concrete mathematical form follow.

Type A: $x^2 = 16$ Type B: $4x^2 = 25$ Type C: $nx^2 = 81$
Type D: $(x + 2)^2 = 36$ Type E: $(x + y)^2 = 49$

Figure 1: Linear equations test

DIRECTIONS

Solve these linear equations.
Show all your work on the work paper.
Write all you answers on the answer paper.

I. Solve for x:

1. $x + 7 = 5$
2. $x - 9 = 10$
3. $4x = 12$
4. $\frac{x}{2} = 8$

II. Solve for x:

5. $x + A = 8$
6. $x - K = 17$
7. $Nx = 9$
8. $\frac{x}{T} = M$

III. Complete the statement.

9. If $x + 3 = 11$, then $x =$ _____.
10. If $x - 14 = 9$, then $x =$ _____.
11. If $5x = 35$, then $x =$ _____.
12. If $\frac{x}{9} = 2$, then $x =$ _____.

IV. Complete the statement.

13. If $x + B = K$, then $x =$ _____.
14. If $x - T = C$, then $x =$ _____.
15. If $Ax = J$, then $x =$ _____.
16. If $\frac{x}{Y} = Z$, then $x =$ _____.

Figure 2: Quadratic equations test

Test: Solve quadratic equations by taking the square root.

DIRECTIONS: 1. Show all your work on the work paper.
2. Write your answers on the answer paper.

I. Solve for x:

1. $x^2 = 16$
2. $4x^2 = 25$
3. $nx^2 = 81$
4. $(x+2)^2 = 36$
5. $(x+y)^2 = 49$

II. Solve for x:

6. $x^2 = a$
7. $25x^2 = R$
8. $nx^2 = c$
9. $(x+1)^2 = d$
10. $(x+y)^2 = k$

III. Complete the statement.

11. If $x^2 = 25$, then x = _____.
12. If $9x^2 = 49$, then x = _____.
13. If $nx^2 = 64$, then x = _____.
14. If $(x+1)^2 = 16$, then x = _____.
15. If $(x+y)^2 = 144$, then x = _____.

IV. Complete the statement.

16. If $x^2 = K$, then x = _____.
17. If $4x^2 = M$, then x = _____.
18. If $nx^2 = J$, then x = _____.
19. If $(x+3)^2 = B$, then x = _____.
20. If $(x+y)^2 = C$, then x = _____.

V. Complete the statement.

21. If $x^2 = a$ and $a>o$, then x = _____.
22. If $16x^2 = K$ and $k>o$, then x = _____.
23. If $nx^2 = C$ and $c>o$, then x = _____.
24. If $(x+1)^2 = m$ and $m>o$, then x = _____.
25. If $(x+y)^2 = d$ and $d>o$, then x = _____.

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